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Origin of energetic cosmic rays III. One-dimensional diffusion from random sources

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Abstract. The model of cosmic ray propagation by one-dimensional diffusion along randomly wandering field lines proposed by Jones in 1971 is developed by considering the effect of random discrete sources on the concentration, anisotropy and mean age of cosmic rays along a line of magnetic flux. We take this mode of propagation to be valid in the energy range 10^{11} eV to 10^{14} eV. The experimental information on the galactic anisotropy, mean age and constancy of the cosmic ray flux is briefly stated. The probability of our being at a time in the earth's cosmic ray history when the fluctuating anisotropy is by chance at its relatively low observed value is evaluated for a number of values of the parameters of propagation.

1. Introduction

A much discussed problem concerning the origin of cosmic rays in the energy region below the break in the primary spectrum at about 10^{15} eV is how to reconcile their relatively short path lengths in the galaxy with their high degree of isotropy. A natural solution is that of Brecher and Burbidge (1972) who propose that the main part of the nuclear component of the cosmic rays even down to the lowest energies is extragalactic in origin. Against this view are arguments concerning the high universal energy density in cosmic rays implied, and estimates that likely sources in the galaxy (supernovae, pulsars, novae) could provide the energy density observed locally. Any model of cosmic ray propagation that leads to low streaming velocities and escape times from the galaxy would allow galactic origin. The propagation model must also be consistent, however, with our knowledge of the properties of the interstellar medium and the galactic magnetic field.

Jokipii and Parker (1969) have shown that although cosmic rays may be tightly bound to magnetic field lines, which on average run parallel to the plane of the galactic disc, the individual field lines will be expected to random walk to the surface of the disc due to turbulent motion of the interstellar gas. Where the field line reaches the surface the cosmic rays may escape. Lingenfelter *et al* (1971) have introduced the concept of compound diffusion. The cosmic rays remain on their own field line, where they propagate by one-dimensional diffusion due to scattering from minor irregularities in the field while the field lines experience three-dimensional random walk with a step size of several tens of parsecs. These workers conclude that a mean free path for the one-dimensional diffusion along a tube, λ, as long as 30 pc is sufficient to reduce the overall anisotropy to the observed level. Allan (1972), however, has emphasized the point made by Ginzburg and Syrovatskii (1964) that the anisotropy observed at the earth is on one particular field line. This, for a given value of λ, is considerably greater than the overall value calculated by Lingenfelter *et al.*

Jones (1971) has used the model of one-dimensional diffusion along a randomly wandering field line and considered the case where cosmic rays are injected continuously and uniformly along the field line. The anisotropy and mean age of the cosmic ray flux seen by an observer at a given point on the field line will be constant in time and dependent solely upon the distances to the two ends, measured along the line. He notes that we do not know the configuration of the field line that contains the earth and derives a probability density for the positions of its end points. For the assumed uniformly distributed sources the relative frequency distribution of streaming velocities (directly related to anisotropy) is obtained. This distribution peaks at zero corresponding to the earth being at the mid-point of its field line. The experimental upper limit to the anisotropy determines the range of positions about the exact mid-point of the line in which the earth may lie. A very low anisotropy is always possible but from the size of this range relative to the length of the field line one may make a judgement of the probability of the earth actually being in the required position.

In the present work we develop this model by considering a rather more physically likely source distribution. Instead of taking a uniform and continuous injection of cosmic rays we take instantaneous points sources (eg novae or supernovae) randomly distributed in space with a poissonian distribution in time interval. At a fixed point on the line the properties of the cosmic ray flux (concentration, mean age and anisotropy) will vary with time. For given values of the diffusion parameters and frequency of sources one may then calculate, for instance, the fraction of the time during which the anisotropy of the flux is less than or equal to the present day observed upper limit. From the size of this fraction one may make judgement of how reasonable is the choice of propagation parameters.

In the model for propagation that we describe in § 3 no energy dependence has been introduced. Because of the effects of the interplanetary magnetic field there can be no measurement of the galactic anisotropy below 10^{11} eV. For comparison with experimental data on anisotropy our model need only refer to particles above this energy. At lower energies hydromagnetic waves are generated by the streaming of the cosmic rays through the interstellar gas. These waves scatter the particles and restrict their streaming velocity to the Alfven velocity. Kulsrud and Cesarsky (1971) have considered the damping of these waves due to collisions between the charged and neutral particles of the interstellar gas. The energy density in cosmic rays above 10^{11} eV (about 5% of that in all cosmic rays) is so low, however, that the growth rate of the waves is much less than the damping rate in a gas with density equal to that in the galactic disc. Thus cosmic rays of the energy we are considering will not be scattered in the disc region by cosmic ray generated waves. We must propose that irregularities causing the scattering are due to some externally produced general turbulence in the interstellar gas.

The one-dimensional diffusion model will break down at sufficiently high energies that the cosmic ray particles are no longer strongly bound to the field lines. Motion of charged particles normal to the magnetic field would be due to gradient or curvature drifts. Consider the gradient drift of a particle moving a distance λ along the field line between successive scatters. The distance drifted during the time will be approximately

$$(E/10^{11}\ \mathrm{eV})(\nabla B/B)(\lambda/6)10^{-4}\ \mathrm{pc}$$

where $(\nabla B/B)$ is the relative gradient of the magnetic field perpendicular to the field line. A field strength of 3 μG is assumed and lengths are in units of parsecs. The direction of drift is mutually perpendicular to the field line and field gradient. The direction of the gradient will vary with position along the field line. If the direction changes randomly at least every step λ along the field line the net distance drifted as the particle diffuses a distance L along the field line will be no more than

$$(E/10^{11}\ \mathrm{eV})(\nabla B/B)(L/6)10^{-4}\ \mathrm{pc}.$$

For example a proton of energy 10^{12} eV in diffusing the whole length of a 3 kpc field line would drift a net distance of 0·5 pc if $(\nabla B/B)$ were as large as 1 pc^{-1}. Field gradients caused by turbulent motion of the interstellar gas are unlikely to be as large as this. Drift due to field line curvature could be comparable to the gradient drift only if the field lines had frequent bends with radius less than about 1 pc. This does not agree with the picture of field line wandering discussed in § 3.2. The energy at which one-dimensional propagation breaks down depends not only upon the net drift but also upon the rate of separation of field lines in the interstellar field (Jokipii 1973). More information on the structure of the field is needed but by 10^{14} eV, where the radius of curvature of a proton in a 3 μG field is 0·03 pc, considerable transfer from one field line to another may be occurring over the lifetime of cosmic rays. It seems more reasonable above this energy to turn to the propagation model developed in paper I (Bell *et al* 1974).

2. Observations

In this section we summarize briefly the observational evidence on anisotropy of the cosmic rays, their age and constancy of concentration. One should note the fact that data on each of these are available only over restricted ranges of energy.

2.1. Anisotropy

An anisotropy in the primary cosmic ray flux with respect to galactic coordinates would appear to an observer on the earth as a regular variation of the secondary intensity with a period of one sidereal day. The observed anisotropy is defined as

$$\delta = (I_{\mathrm{max}} - I_{\mathrm{min}})/(I_{\mathrm{max}} + I_{\mathrm{min}}).$$

In practice a first harmonic sinusoidal variation of intensity with sidereal time is fitted to the data and I_{max} and I_{min} are the maximum and minimum secondary particle intensities from this fit. Figure 1 shows values of δ from a number of experiments plotted at the estimated median energy of the primary cosmic rays producing the signal. The two lowest energy points are from measurements of muons underground; the remainder are from observations of air showers. The error bars and upper limits indicate one standard deviation. The curve shows our estimate of the upper limit to the anisotropy as a function of energy. The relatively large anisotropy reported by Delvaille *et al* (1962) which is at variance with the results of Cachon (1962) and Kolomeets *et al* (1969) in the same energy range has not been confirmed by more recent observations. The lowest energy point, that of Elliott *et al* (1970), apparently gives the most stringent limit to the anisotropy. It is obtained from the variation in intensity of muons which penetrate a depth of $6 \times 10^3\ \mathrm{g\,cm^{-2}}$ of rock. We have plotted it at a median primary energy of $1{\cdot}5 \times 10^{11}$ eV which is obtained using recent accelerator data on multiplicities of secondary particles produced in proton–nucleus interactions. Particles of this energy may be influenced by the interplanetary field which will tend to reduce any galactic

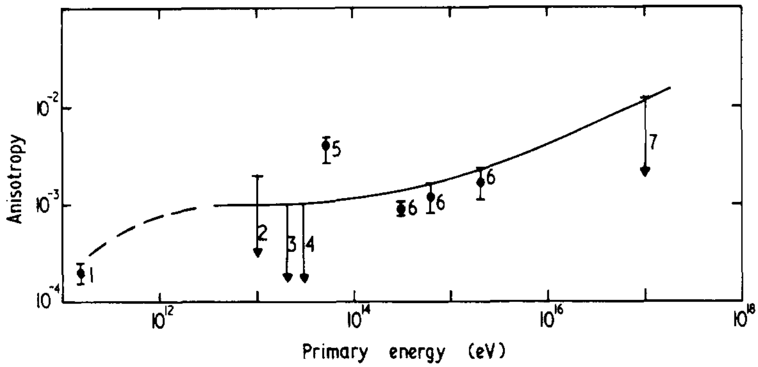

Figure 1. Experimental measurements of cosmic ray anisotropy: 1, Elliot *et al* (1970); 2, Sherman (1953); 3, Cachon (1962); 4, Kolomeets *et al* (1969); 5, Delvaille *et al* (1962); 6, Daudin *et al* (1956); 7, Lapikens *et al* (1971).

anisotropy. If a model of the interplanetary field is chosen the effect on the anisotropy as a function of energy can be calculated. For instance Barnden and McCracken (1973) have made calculations for 2-sector and 4-sector models and concluded that a detector at 6×10^3 g cm^{-2} would record an anisotropy typically 2·5 times smaller than the true galactic anisotropy. There may be enough uncertainty, however, in the response function of the detector to primary energy and the form of the interplanetary field that 2×10^{-4} represents the actual upper limit to the galactic anisotropy at $1{\cdot}5 \times 10^{11}$ eV. A measurement at two or three times higher primary energy would be sure of avoiding any diluting effect of the interplanetary field. We conclude that in the region of primary energy between 10^{11} eV and 10^{14} eV the upper limit to the galactic anisotropy lies between 10^{-4} and 10^{-3}. For the purposes of our calculations we take three values of the limiting anisotropy, $\delta = 10^{-4}$, $\delta = 4 \times 10^{-4}$ and $\delta = 10^{-3}$.

2.2. *Age of the cosmic rays*

The age distribution of cosmic rays reaching the earth can be deduced from detailed measurements of the charge composition of cosmic rays. Knowledge of the proportion of secondary Li, Be and B in the cosmic ray flux produced by the spallation mainly of C, N and O on the interstellar gas, and the proportion of nuclei with $17 < Z < 25$ produced by spallation of iron, allows one to deduce the path length in terms of the total amount of matter traversed between the source and the earth. Shapiro *et al* (1970) have shown that a unique path length is not in agreement with the data. They conclude that an exponential path length distribution with a mean path of 6 g cm^{-2} gives the best fit. Other distributions which roughly approximate to the exponential one give almost as good fits, however.

These results are derived from measurements of the composition in the region of 10^9 eV/nucleon. Recently a number of experiments have extended the measurements beyond 10^{11} eV. Ramaty *et al* (1973) have given a summary. All of the observations indicate that the proportion of secondary nuclei is decreasing with energy. The precise amount of change is still rather uncertain but at the highest energies observed (4×10^{10} eV/nucleon) the path length has decreased to $2 \pm 1{\cdot}5$ g cm^{-2}. Because of the uncertainties it is not possible to make a meaningful extrapolation to the region beyond

10^{11} eV. One may remark, however, that it is unlikely that the path length should increase with energy beyond the measured region so one could treat the value of $2 \pm 1{\cdot}5$ g cm^{-2} as an upper limit for energies beyond 10^{11} eV.

To convert the amount of material traversed into an age for the cosmic rays one needs to know the mean density of the interstellar gas through which they have travelled assuming, as is conventional, that the spallation takes place almost entirely in the interstellar gas and not in the sources. An average density for the interstellar material in the galaxy of one hydrogen atom per cubic centimetre leads to an age of $1{\cdot}2 \times 10^6$ yr for the traversal of 2 g cm^{-2} of gas. The interstellar material is not uniform in density, however. If the field line on which the earth lies happens to be mainly in intercloud regions the mean density could be as low as 0·15 hydrogen atoms per cubic centimetre and the cosmic ray age would be correspondingly longer.

2.3. *Constancy of the cosmic ray flux*

Measurements of the abundances of radioactive isotopes formed by the interactions of cosmic rays in meteorites give some indication of time variations in the cosmic ray flux. They appear to show that the present day value of the flux is within a factor of two of the flux averaged over the last 10^5 to 10^7 yr (Geiss 1963). The energy region of the cosmic rays producing most of the transmutations observed is 10^9 to 10^{10} eV, ie a much lower energy range than that which we are considering. In our calculations we do, however, look at the effect of including this constraint on the fluctuations in the cosmic ray flux.

3. The model of cosmic ray propagation

3.1. *Solution of the one-dimensional diffusion equation*

Along a field line the cosmic rays experience one-dimensional diffusion. The diffusion equation is

$$\frac{\partial N}{\partial t} = D\frac{\partial^2 N}{\partial x^2}$$

where N is the concentration, D the diffusion coefficient and x is the displacement measured along the line. We assume that the cosmic rays freely escape when the field line reaches the boundary of the disc. The boundary conditions are therefore $N = 0$ at $x = 0$ and $x = h$, for all t, where h is the length of the field line. For a source which is instantaneous in time at $t = 0$ and is presented by $S\,\delta(x-x_0)$ where S is the source strength, which we take the same for all sources, a solution of the diffusion equation is

$$N(x, t) = \frac{2S}{h}\sum_{n=1}^{\infty} \exp\left(\frac{-n^2\pi^2 Dt}{h^2}\right) \sin\left(\frac{n\pi x_0}{h}\right) \sin\left(\frac{n\pi x}{h}\right).$$

This series converges rapidly for $t > h^2/4\pi D$. For $t < h^2/4\pi D$ the solution is

$$N(x, t) = \frac{S}{(4\pi Dt)^{1/2}}\left\{\exp\left(-\frac{(x_0-x)^2}{4Dt}\right) - \left[\exp\left(-\frac{(-x_0-x)^2}{4Dt}\right) + \exp\left(-\frac{(2h-x_0-x)^2}{4Dt}\right)\right]\right.$$
$$\left. + \left[\exp\left(-\frac{(2h+x_0-x)^2}{4Dt}\right) + \exp\left(-\frac{(-2h+x_0-x)^2}{4Dt}\right)\right] + \ldots\right\}.$$

The flux is $J = -D\,\partial N/\partial x$ and the one-dimensional anisotropy is given by J/Nv. The velocity v is the magnitude of the effective mean velocity of cosmic rays parallel to the field line. The observed distribution of arrival directions of particles at a point on the line will in fact be three dimensional rather than one dimensional because the particles are distributed in pitch angle. Assuming that scattering along the field lines causes complete mixing of pitch angles, $v = c/3$. The anisotropy is $\delta = 3J/Nc$.

For a number of sources occurring at x_i and t_i the individual concentrations N_i and fluxes J_i can be summed and the resultant concentration, anisotropy and mean age $\bar{t} = \Sigma_i N_i(t-t_i)/\Sigma_i N_i$ can be calculated. To do this a set of sources was generated, randomly distributed in space along the field line and following a poissonian distribution of time intervals with an average time τ_s/h between sources.

3.2. Distribution in length of field lines

The length of the field line passing through the earth and the position of the earth relative to its ends are both unknown. A probability distribution of the lengths of field lines must therefore be calculated. To do this we took the stochastic model of the interstellar field of Jokipii and Parker (1969). The magnetic field lines random walk due to their being carried bodily by the turbulent motion of the gas. The gas motions are considered to be uncorrelated after a distance in the galactic plane of 100 pc. We then made the simplification that after each 100 pc step in the plane the line has been displaced either up or down a distance of 45 pc. Since the earth is near to the mid-plane of the galaxy we demanded that the field line passed through the mid-plane. Using a Monte Carlo approach, we calculated the distribution of lengths to reach the boundaries of the disc, 130 pc above and below the mid-plane. This length distribution, which we shall call distribution I, is shown in figure 2. The value of the vertical displacement of the field line after each step is estimated from the distribution of vertical interstellar gas velocities and the rate of change of gravitational acceleration with distance from the galactic plane.

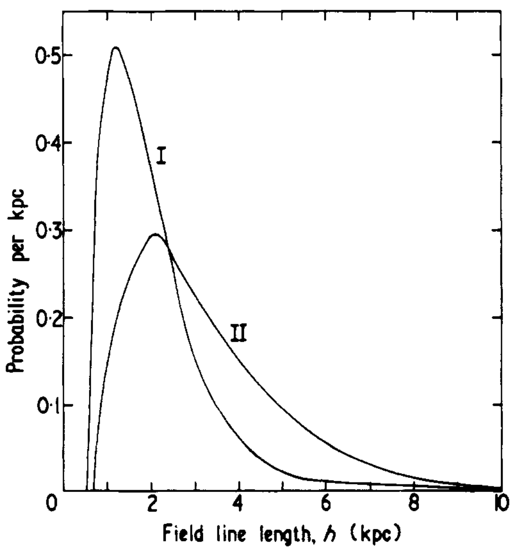

Figure 2. Length distribution of magnetic field lines in the galactic disc for two field models.

Because the estimated value is rather uncertain we generated distribution II, also shown in figure 2, using an alternative vertical displacement of 35 pc.

3.3. *Interval between sources*

In order to proceed further, a value has to be adopted for the average time interval between sources. Possible instantaneous point sources of cosmic rays are novae and supernovae which are estimated to occur in the galaxy at rates of about 260 yr^{-1} (Sharov 1972) and one per 26 yr (Tamman 1970) respectively. To convert these to intervals between sources on a particular field line one must have an estimate of the effective cross sectional area surrounding a field line from which there is input of cosmic rays from sources, ie how close to the line which passes through the earth a source must lie in order that we should be able to observe its cosmic rays.

In the first few years after a supernova explosion the magnetic field in the remnant will be sufficiently high that cosmic rays will be trapped. As the remnant expands its magnetic field decreases and the cosmic rays will eventually escape and become attached to an interstellar field line. The problem of transfer of cosmic rays from the remnant to the galaxy as a whole is a complex one. The effective radius of the source at the stage that cosmic rays are transferred is a necessary parameter of the present computation. The path length (in $g\,cm^{-2}$) of material traversed in the remnant is also of interest. If it is a significant fraction of the total it may provide an explanation of the observed energy dependence of path length referred to in § 2.2. Ostriker and Gunn (1971) have developed a model for the expansion of a supernova remnant with continuous energy input from a pulsar. For their 'standard' type 2 supernova the magnetic field falls to a value about twice that of the galactic magnetic field after 3000 yr. The radius of the remnant is then 10 pc. As an upper limit to the effective source radius one can take the radius of the remnant when it merges into the interstellar medium. This occurs when the expansion velocity falls to the 10 $km\,s^{-1}$ turbulent velocity of the interstellar gas. The radius is then about 30 pc. Assuming that supernovae are distributed uniformly in the galactic disc of radius 15 kpc and thickness 260 pc, values of τ_s = (average time interval between sources on a line) × (length of field line) of $1{\cdot}4 \times 10^7$ and $1{\cdot}6 \times 10^6$ yr kpc are obtained for effective source radii of 10 pc and 30 pc respectively. Taking into account the age of cosmic rays, a few sources would be contributing to the flux at any given time.

The frequency of novae and their energy release are high enough that they must also be considered as possible sources of cosmic rays although their ability to accelerate particles in the energy range above 10^{11} eV is in doubt. In the absence of information on the confinement of particles in the expanding nova shell one can quote only an upper limit to the effective source radius corresponding to the shell merging with the interstellar medium. For a shell of 10^{-4} solar masses having initial velocity of 1000 $km\,s^{-1}$ this radius is 0·4 pc, giving $\tau_s = 1{\cdot}4\ 10^6$ yr kpc. In the present work in order to show the effect of variation of interval between sources we performed calculations for two values, $\tau_s = 4 \times 10^6$ yr kpc and $\tau_s = 2 \times 10^7$ yr kpc.

4. Results of the calculations

The concentration of cosmic rays, the anisotropy of the flux, and their mean age were calculated at given positions, on lines of given length, at successive steps in time. The steps in time were made small compared to the mean interval between sources. The two values of τ_s, given in § 3.3, were used. To find the effect of varying the diffusion coefficient D three values were used: $0{\cdot}102\ pc^2\ yr^{-1}$, $0{\cdot}307\ pc^2\ yr^{-1}$ and $1{\cdot}02\ pc^2\ yr^{-1}$. These correspond to diffusion mean free paths λ of 1 pc, 3 pc and 10 pc, taking the mean component of the velocity along the field line to be $c/3$ due to pitch angle scattering. There is no real observational information on the value of λ. In paper I it is shown that, in the higher energy region considered there, a reasonable value for the mean free path for three-dimensional diffusion is 20 pc but there is no compelling reason for equating this to the mean free path for one-dimensional diffusion along a flux tube. We made our calculations for a range of values of λ that we expected to give a reasonable probability of a low anisotropy without giving too high a mean age for the cosmic rays.

Jokipii (1971) has put forward an argument against a mean free path of less than about 1 pc. We have previously remarked that any waves that scatter cosmic rays above 10^{11} eV in the disc region must be generated by turbulent motion of the gas. Energy will thus be transferred from the gas to the cosmic rays: the cosmic rays will experience a degree of Fermi acceleration and the waves will be damped. The characteristic time for acceleration is inversely proportional to λ and for $\lambda \simeq 1$ pc it approximates to the age of the cosmic rays. The force of the argument is reduced, however, in that it need only apply to cosmic rays with energy greater than 10^{11} eV. Because they contain only a small proportion of the total cosmic ray energy the required energy input to the waves from the gas need not be excessive.

Figure 3 shows an example of the time variation of the concentration, mean age and anisotropy of cosmic rays. This particular run of values was obtained at the mid-point of a field line 3 kpc long for the case $\lambda = 3$ pc and $\tau_s = 4\times10^6$ yr kpc. Many similar runs were generated and a record was kept of the fraction of the time that the anisotropy was less than 10^{-4}, 4×10^{-4} and 10^{-3}. This fraction can be equated to the probability of an observer at that position on the line seeing an anisotropy less than the stated value.

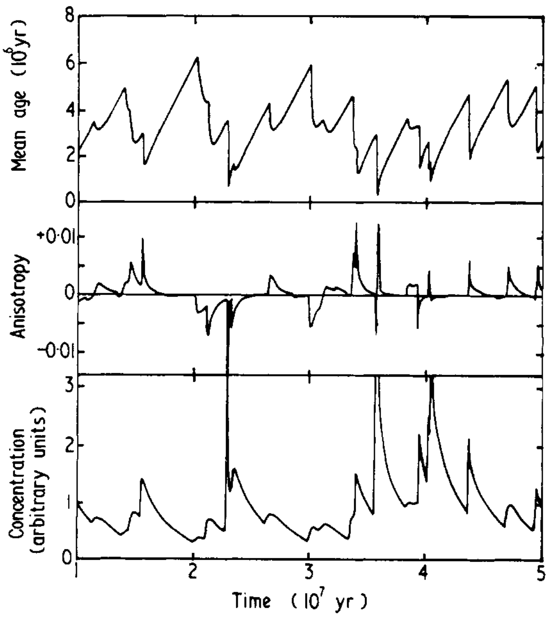

Figure 3. Example of time variation of concentration, mean age and anisotropy of cosmic rays at the mid-point of a field line of length 3 kpc.

In figures 4, 5 and 6 the probability is plotted as a function of field line length h and relative position x/h along the line for various values of τ_s and D. As expected the probability of seeing only a very small anisotropy is, in general, highest for the longest field lines. Comparison of figures 4 and 6 shows that an increase in D leads to a lower probability of seeing $\delta < 10^{-3}$ at all positions on all lines. Comparison of figures 5 and 6 shows that, on the shorter lines, an increased frequency of sources leads to somewhat higher probability of $\delta < 10^{-3}$.

A check was also made on the constancy of the cosmic ray concentration. A record was kept of the probabilities (ie fraction of time) that the instantaneous concentration is within a factor of two of the concentration averaged over the previous 10^5, 10^6 and 10^7 yr. These probabilities are in general greater than or equal to the probability that $\delta < 10^{-3}$. There is in addition a strong correlation between the anisotropy and concentration.

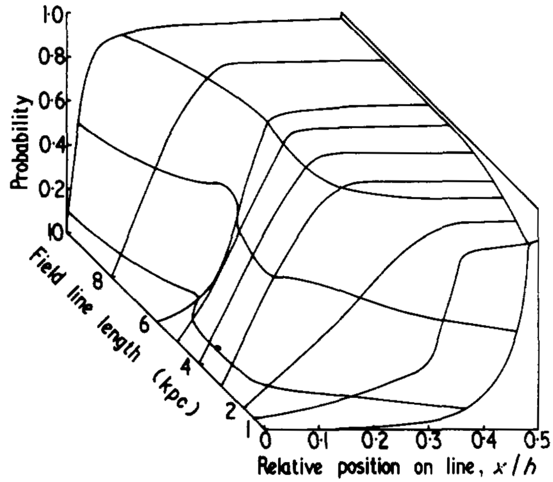

Figure 4. Probability that $\delta < 10^{-3}$ as a function of field length and position on the line for $\lambda = 1$ pc and $\tau_s = 4 \times 10^6$ yr kpc^{-1}.

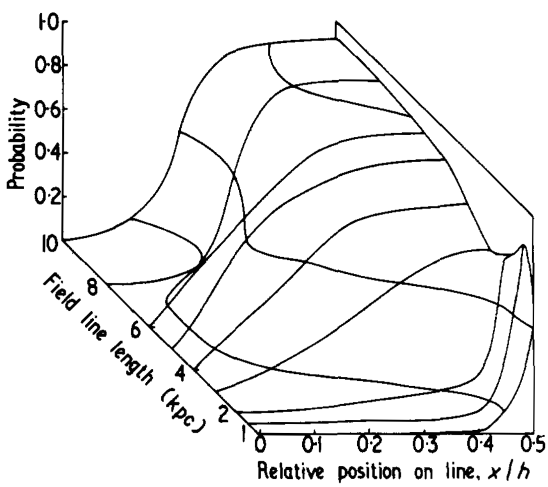

Figure 5. Probability that $\delta < 10^{-3}$ for $\lambda = 3$ pc and $\tau_s = 4 \times 10^6$ yr kpc^{-1}.

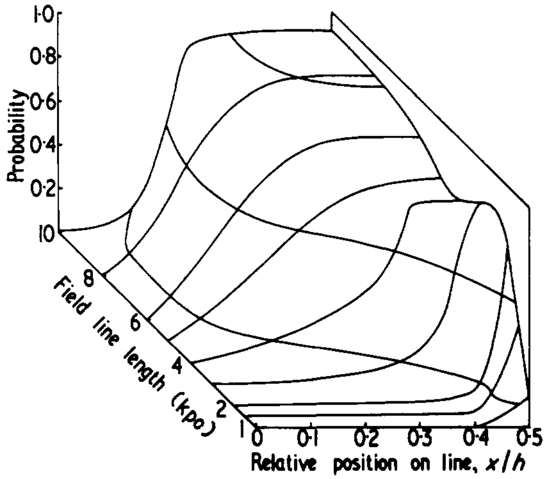

Figure 6. Probability that $\delta < 10^{-3}$ for $\lambda = 3$ pc and $\tau_s = 2 \times 10^7$ yr kpc^{-1}.

The occurrence of a nearby source will simultaneously increase the concentration and anisotropy. The relatively low observed limit to the anisotropy is a more stringent condition than the rather imprecise limit on the variation of the concentration (which in any case applies to much lower energy particles than we are considering). We assume that this condition for constancy of concentration will always be met when the anisotropy is as low as the observed value.

In order to find the overall probability of observing anisotropies less than the upper limiting values we first assume that there is equal probability of the earth being at any position along the field line and average over the length of the line. We then fold in these probabilities, as a function of length, to the field line length distributions. Table 1 gives the probabilities for the three anisotropy levels for the twelve combinations of field line length distribution, D and τ_s.

In order to see the effect of our having considered discrete sources instead of a uniform distribution, the corresponding probabilities were calculated for this latter case too. In this case the anisotropy is a unique function of the distance, $x - \frac{1}{2}h$, from the mid-point

Table 1. Probabilities of observing given anisotropies for the case of discrete sources for various values of D (pc^2 yr^{-1}) and τ_s (yr kpc).

Field line distribution	δ	$D = 0.102$		$D = 0.307$		$D = 1.02$	
		$\tau_s = 4 \times 10^6$	$\tau_s = 2 \times 10^7$	$\tau_s = 4 \times 10^6$	$\tau_s = 2 \times 10^7$	$\tau_s = 4 \times 10^6$	$\tau_s = 2 \times 10^7$
I	$< 10^{-3}$	0·43	0·34	0·16	0·086	0·046	0·020
I	$< 4 \times 10^{-4}$	0·24	0·14	0·084	0·056	0·018	0·009
I	$< 10^{-4}$	0·091	0·060	0·038	0·037	0·004	0·002
II	$< 10^{-3}$	0·61	0·54	0·32	0·21	0·11	0·051
II	$< 4 \times 10^{-4}$	0·38	0·29	0·17	0·12	0·044	0·026
II	$< 10^{-4}$	0·12	0·11	0·054	0·060	0·011	0·005

of the line, $\delta = \lambda(2x-h)/(xh-x^2)$. The probability of observing an anisotropy less than δ can then be equated to the fraction of the total length of the line within which this condition is satisfied so the probability is

$$P = [1+(2\lambda/\delta h)^2]^{1/2} - 2\lambda/\delta h.$$

These values are folded into the length distributions to give the overall probabilities in table 2.

Table 2. Probabilities of observing given anisotropies for a continuous source distribution for various values of D ($pc^2\ yr^{-1}$).

Field line distribution	δ	$D = 0{\cdot}102$	$D = 0{\cdot}307$	$D = 1{\cdot}02$
I	$<10^{-3}$	0·39	0·16	0·049
I	$<4\times10^{-4}$	0·18	0·065	0·020
I	$<10^{-4}$	0·049	0·016	0·005
II	$<10^{-3}$	0·51	0·24	0·081
II	$<4\times10^{-4}$	0·28	0·10	0·032
II	$<10^{-4}$	0·081	0·027	0·008

The simplifying assumption that the earth can be at any position on its field line will introduce some error. Since the earth is known from astronomical evidence to be near the central plane of the galactic disc it is also more likely to be near the centre, rather than the ends, of its field line, and the shorter the line the more likely it is that the earth is central. As a check, for one set of values of the parameters, the actual probability distribution of the earth's position was folded in. The result was an increase in the overall probability by a factor of 1·25. All of the probabilities listed in tables 1 and 2 should be enhanced by roughly this factor.

The mean age of the cosmic rays at a given position also fluctuates with time as shown in figure 3. The average of these ages is independent of the relative position on the line but varies with the field line length and the values of λ and τ_s. In table 3 values are given of the average age for the various values of λ and τ_s. The value quoted corresponds to the most probable field line length in each case.

Table 3. Average ages of cosmic rays (10^6 yr) for various values of τ_s (yr kpc) and λ.

τ_s	$\lambda = 1$ pc	$\lambda = 3$ pc	$\lambda = 10$ pc
4×10^6	7	4·5	2
2×10^7	11	7	6

The variation of cosmic ray concentration with time shown in figure 3 gives an indication of the expected degree of variation of concentration from one point to another in the galactic disc. At present there are no observations of the proton component to support or refute this. If we are by chance at a period when the anisotropy is low, the local concentration will also be at a relatively low value, about half of the average value throughout the disc. Comparison of the amount of synchrotron radiation from the disc with the measured electron energy spectrum suggests that the local electron component of cosmic rays is indeed somewhat lower than the average.

5. Conclusions

Comparison of the probabilities in tables 1 and 2 shows that, for the same path length distributions and values of D, they are within a factor of two. Thus the assumption of random discrete sources, when the interval between sources is of the order of 10^7 yr kpc, does not give a very different result from that for the much simpler assumption of a uniform source distribution. This is a little surprising since the interpretation of the probability is different in the two cases. In the former case it is the chance that we are at a point in the earth's cosmic ray history when the fluctuating cosmic ray anisotropy is at a low value; in the latter case it is the chance that we happen to be situated near enough to the centre of our flux tube. For short lines, where the overall probability is low the uniform source distribution gives 100% probability over a narrow region of the field line centred on its mid-point while the random sources give a much lower probability per unit length spread over a correspondingly larger fraction of the field line.

The field line length distribution II gives higher probabilities than I since it favours longer lines, which in turn lead to lower anisotropies. Similarly a smaller value of λ gives lower anisotropies and higher probabilities. A constraint on lowering λ is imposed by the limit to the age of the cosmic rays. The figures in table 3 show that even for $\lambda = 3$ pc, which gives a reasonably high probability of a low anisotropy, the age is rather greater than that deduced in § 2.2, although it should be emphasized again that the age estimates are approximate and relate to energies lower than those for which the anisotropy is measured. Increasing λ to 10 pc brings the ages closer to observation but it can be seen from table 1 that this reduces the anisotropy probabilities considerably. It is interesting to note that the smaller value of τ_s gives both a higher probability of the anisotropy criterion being met and a lower age. The propagation of cosmic rays close to their sources needs further investigation because it determines the relationship of the frequency of sources in the galaxy as a whole to the effective frequency on individual field lines.

Although the observed properties of the cosmic ray flux may well be the result of our being at a particular time in the earth's cosmic ray history, the probabilities given in table 1 are rather small if the anisotropy is indeed less than 10^{-4} with $\lambda > 3$ pc. Anisotropies will be reduced if there is a finite probability of the cosmic rays being reflected at the 'ends' of the field line rather than freely escaping as assumed in the present work. Skilling (1971) has shown that at sufficient heights above the galactic plane the gas density will be low enough that hydromagnetic waves generated by cosmic rays above 10^{11} eV can propagate and in turn scatter the cosmic rays. He assumes that the cosmic rays propagate freely with no scattering in the more dense parts of the disc. At the boundary between the free zone and the wave zone reflection may occur. Holmes (1974) has pointed out that the energy dependence of the height of this boundary can give an explanation of the energy dependence of path length of cosmic rays. In the present work we take it that cosmic rays are scattered in the disc region also. Provided that the self-generated waves cause a decrease in the effective diffusion coefficient at the boundary of the disc reflection will still occur.

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